# Message passing resource allocation for the uplink of multicarrier systems

Andrea Abrardo*, Paolo Detti*, Marco Moretti†

*Dipartimento di Ingegneria dell'Informazione, University of Siena, Italy
†Dipartimento di Ingegneria dell'Informazione, University of Pisa, Italy

*Abstract*—We propose a novel distributed resource allocation scheme for the up-link of a cellular multi-carrier system based on the message passing (MP) algorithm. In the proposed approach each transmitter iteratively sends and receives information messages to/from the base station with the goal of achieving an optimal resource allocation strategy. The exchanged messages are the solution of small distributed allocation problems. To reduce the computational load, the MP problems at the terminals follow a dynamic programming formulation. The advantage of the proposed scheme is that it distributes the computational effort among all the transmitters in the cell and it does not require the presence of a central controller that takes all the decisions. Numerical results show that the proposed approach is an excellent solution to the resource allocation problem for cellular multi-carrier systems.

## I. INTRODUCTION

Orthogonal Frequency Division (OFDM) modulation is one of the candidate technologies for future generation broadband wireless networks. Provided that the system parameters are accurately dimensioned, OFDM transmissions are not affected by intersymbol interference (ISI) even in highly dispersive channels. Moreover, OFDM can effectively exploit the channel frequency diversity [1], [2] by dynamically adapting power and modulation format on all subcarriers. Orthogonal frequency multiple access (OFDMA) is the multiple access scheme based on OFDM: each user is allocated a different subset of orthogonal subcarriers. When the transmitter possesses full knowledge of channel state information, the subcarriers can be allocated according certain optimality criterion to increase the overall spectral efficiency, exploiting the so-called *multiuser diversity*. Resource allocation is one of the most efficient techniques to increase the performance of multicarrier systems. In fact, propagation channels are independent for each user and thus the sub-carriers that are in a deep fade for one user may be good ones for another. Many resource allocation algorithms have been designed for taking advantage of both the frequency selective nature of the channel and the multi-user diversity. In most cases dynamic resource allocation has been formulated with the goal of either minimizing the transmitted power with a rate constraint [3], [4] or maximizing the overall rate with a power constraint [5], [6].

In this paper, starting from the formulation of resource allocation problem as a minimization problem, we propose a novel distributed resource allocation scheme for the up-link of a cellular multi-carrier system based on the message passing (MP) algorithm. MP algorithms have gained their momentum in the last years owing to their broad usage in LDPC and turbo channel decoding applications [7]. In this setting, messages represent probabilities or beliefs [1] , which are exchanged with the goal of achieving an optimal bit decisions. We will show that resource allocation may rely on a similar MP procedure: with the goal of achieving a global optimal assignment, each transmitter iteratively sends and receives information messages to/from the base station until an allocation decision is taken. The exchanged messages are the solution of small distributed allocation problems. To reduce the computational load, the MP problems at the terminals follow a dynamic programming formulation. The advantage of the proposed scheme is that it distributes the computational effort among all the transmitters in the cell and it does not require the presence of a central controller for a problem that in its original formulation is NP-hard, as pointed out at the end of Section II.

The rest of the paper is organized as follows. In Section II we describe the system model. In Section III we show how message passing can be tailored to solve the problem of resource allocation in the uplink of a cellular system. In Section IV we present simulation results. Finally, in Section V we discuss future work and draw our conclusions.

## II. SYSTEM MODEL

We focus on the problem of channel allocation for the uplink of an OFDMA system. The overall frequency bandwidth is divided into orthogonal sub-carriers and, to reduce allocation complexity, we group sets of adjacent subcarriers into $F$ *subchannels*. As long as the bandwidth spanned by a subchannel is smaller than the channel coherence bandwidth, the channel spectrum can be approximated as flat in the subchannel. Thus, we can assume that the choice of performing resource allocation on subchannels rather than on subcarriers causes almost no loss in diversity.

Allocation is performed with the goal of minimizing the overall transmitted power subject to rate constraints per user. Due to practical considerations, we consider only a limited set $\mathcal{Q} = \{0, \ldots, Q\}$ of possible transmission formats. A given transmission format $q$ corresponds to the usage of a certain error correction code and symbol modulation that leads to a spectral efficiency $\eta_q$: a user employing format $q$ on a certain subchannel transmits with rate $R = B\eta_q$, $B$ being the bandwidth of each subchannel. The spectral efficiency

---

[1]the algorithm is also known as the belief propagation algorithm

associated with format $q = 0$ is $\eta_0 = 0$, i.e. no transmission at all. The target SNR to achieve the spectral efficiency $\eta_q = \log_2(1 + SNR(q))$ is $SNR(q) = 2^{\eta_q} - 1$. Let $\mathcal{F}$ be the set containing the $F$ available subchannnels. Given the format $q$, the power $P_{n,f}(q)$ necessary to user $n$ to transmit on subchannel $f$ is computed as

$$P_{n,f}(q) = SNR(q)\frac{BN_0}{|H_{n,f}|^2} \tag{1}$$

where $H_{n,f}$ is the channel gain between user $n$ and the BS on the $f$-th link and $N_0$ is the power spectral density of the zero-mean thermal noise. Channel assignment is exclusive: each subchannel can be assigned to only one user and with a just a single format.

Our resource allocation problem is a constrained minimization problem in the vector $\mathbf{x} = [x_{1,1}, \ldots, x_{N,F}]$, where the variable $x_{n,f} \in \mathcal{Q}$ indicates the modulation format of user $n$ on subchannel $f$. The allocation problem has the following general form

$$\text{minimize } f_0(\mathbf{x}) \tag{2}$$
$$\text{subject to}$$
$$d_f(\mathbf{x}) \leq 1 \qquad f \in \mathcal{F} \tag{C1}$$
$$h_n(\mathbf{x}) \geq b_n \qquad n = 1, \ldots, N \tag{C2}$$
$$g_n(\mathbf{x}) \leq P_{max,n} \qquad n = 1, \ldots, N \tag{C3}$$

Here the objective function $f_0 : \mathcal{D} \to \mathbb{R}_+$ is the cost in terms of overall power of the allocation $\mathbf{x}$:

$$f_0(\mathbf{x}) = \sum_{n=1}^{N} \sum_{f \in \mathcal{F}} P_{n,f}(x_{n,f}) \tag{3}$$

the domain $\mathcal{D} = \mathcal{Q}^{NF}$ is the set of all possible transmission formats on all subchannels for all users. The inequality constraints functions $d_f : \mathcal{D} \to \mathbb{R}_+$ represent the condition of exclusive allocation for all subchannels

$$d_f(\mathbf{x}) = \sum_{n=1}^{N} \mathcal{I}(x_{n,f}) \tag{4}$$

where $\mathcal{I}(x_{n,f})$ is 1 if $1 \leq x_{n,f} \leq Q$ and 0 otherwise. The constraints functions $h_n : \mathcal{D} \to \mathbb{R}_+$ enforce that each user transmits at least with rate $b_n$

$$h_n(\mathbf{x}) = \sum_{f \in \mathcal{F}} B\eta_{x_{n,f}} \tag{5}$$

The constraints functions $g_n : \mathcal{D} \to \mathbb{R}_+$ enforce that each user does not exceed its maximum transmitting power $P_{max,n}$

$$g_n(\mathbf{x}) = \sum_{f \in \mathcal{F}} P_{n,f}(x_{n,f}) \tag{6}$$

The radio resource allocation problem introduced above can be shown to be NP-hard by a straightforward reduction from the NP-hard problem *Multiprocessor Scheduling* [8], even when only one single transmission format is considered.

## III. RESOURCE ALLOCATION VIA MESSAGE PASSING

In the following, we formulate the allocation problem in such a way that can be solved with a message passing technique (MP). The advantage of MP is that the computation load is distributed among the various nodes by locally passing simple messages among simple processors whose operations lead, after some time, to the solution of a global problem.

First of all, to simplify the allocation task we assume that each user selects a subset of all available subchannels. Let $\mathcal{P}_n \subset \mathcal{F}$ be the subset of cardinality $P < F$ of subchannels that can be allocated to user $n$. In other terms, we assume that $x_{n,f}$ may be different from zero only if $f \in \mathcal{P}_n$. As for the choice of the subchannels in $\mathcal{P}_n$, we make the natural assumption that they represent the $P$ best subchannels for user $n$, i.e. $\mathcal{P}_n = \{f \in \mathcal{F} : |H_{n,f}| \text{ is one of the } P \text{ largest values for user } n\}$. Each user may pre-compute its subset of channels before the resource allocation algorithm is initiated [2].

For our scope, it is convenient to interpret the resource allocation problem as a minimum cost problem, where the unfulfillment of constraints in (2) gives an infinite cost. Thus, we take care of the constraints C1 by introducing the cost function $C(f)$ ($f \in \mathcal{F}$), which is 0 if the exclusive requirement on subchannel $f$ is fulfilled and $\infty$ otherwise

$$C(f) = \begin{cases} 0 & \text{if } \sum_{n \in \mathcal{N}(f)} \mathcal{I}(x_{n,f}) \leq 1 \\ \infty & \text{otherwise} \end{cases} \tag{7}$$

where $\mathcal{N}(f)$ is the subset of users that might use subcarrier $f$, i.e. $\mathcal{N}(f) = \{n : f \in \mathcal{P}_n\}$. The constraints C2 and C3 are dealt by introducing the set of functions $W(n)$ ($n = 1, \ldots, N$), defined as:

$$W(n) = \begin{cases} \sum_{f \in \mathcal{P}_n} P_{n,f}(x_{n,f}) & \text{if } \sum_{f \in \mathcal{P}_n} B\eta_{x_{n,f}} \geq b_n \\ & \sum_{f \in \mathcal{P}_n} P_{n,f}(x_{n,f}) \leq P_{max,n} \\ \infty & \text{otherwise} \end{cases} \tag{8}$$

Despite notation complexity, the meaning of (8) is straightforward: $W(n)$ is the power transmitted by user $n$ if power and rate constraints for user $n$ are fulfilled, and $\infty$ otherwise. Given the above, it is straightforward to rewrite the resource allocation problem in (2) as:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{arg\,min} \left( \sum_{f \in \mathcal{F}} C(f) + \sum_{n=1}^{N} W(n) \right). \tag{9}$$

Since the goal is to get a distributed solution for the above minimization problem, we focus on a single variable, e.g., $x_{n,f}$, and rewrite the same problem in a form suited for MP implementation as:

$$\hat{x}_{n,f} = \underset{x_{n,f}}{arg\,min} \left[ \underset{\bar{x}_{n,f}}{min} \left( \sum_{f \in \mathcal{F}} C(f) + \sum_{n=1}^{N} W(n) \right) \right] \tag{10}$$

where notation $\min_{\bar{x}_{n,f}}$ denote the minimum over all variables $\mathbf{x}$ except $x_{n,f}$.

## A. MP implementation

The MP algorithm has been broadly used in the last years in channel coding applications. In particular, when dealing with bitwise MAP channel decoding, MP finds an optimum solution for the sum-product problem, provided that the correspondent factor graph is a tree [9]. The MP algorithm for the sum-product problem derives by the distributive law, i.e., by the property $\sum \prod = \prod \sum$. However, since $\min \sum = \sum \min$, the same property still holds for min-sum problems, where minimization replaces addition in the original formulation[3] and addition replaces multiplication. By exploiting such a formal equivalence, it is straightforward to adapt the MP algorithm to the min-sum problem (10). To elaborate, let associate with problem (10) a factor graph, where variables $x_{i,p}$ are circular nodes and functions $C(f)$ and $W(n)$ are square nodes. Variable nodes are connected with function nodes by an edge if and only if the variable appears in the function, i.e. $x_{n,p}$ is connected to the $P$ functions $C(\ell)$ with $\ell \in \mathcal{P}_n$ and to $W(n)$. The factor graph for (10) is depicted in Fig. 1 where we denote by $\tilde{x}_{n,p}$ $(p = 1, \ldots, P)$ the transmit format for user $n$ on the p-th ordered element of $\mathcal{P}_n$. Following a MP strategy, variable and function nodes exchange messages along their connecting edges until variable nodes can decide on the value of $\tilde{x}_{i,p}$.

Let now assume that the factor graph is a single tree, i.e., a connected graph where there is an unique path to connect two nodes. In this case, the implementation of the MP approach is straightforward. Let firstly introduce messages as $(Q + 1)$-dimensional vectors, denoted by $\mathbf{m} = \{m(0), m(1), \ldots, m(Q)\}$. In particular, denote by $\mathbf{m}_{n,f}^{(CV)}/\mathbf{m}_{n,f}^{(VC)}$ messages exchanged between the $C$ function nodes and the connected variable nodes, and by $\mathbf{m}_{n,f}^{(WV)}/\mathbf{m}_{n,f}^{(VW)}$ messages exchanged between the $W$ function nodes and the connected variable nodes.

As in the classical sum-product scenario, message passing starts at the leaf nodes, i.e., those nodes which have only one connecting edge. In particular, each variable leaf node passes an all-zero message to its adjacent function node, whilst each function leaf node passes the value of the function to its adjacent node. After initialization at leaf nodes, for every node we can compute the outgoing message as soon as all incoming messages along all other connected nodes are received.

As far as variable nodes are of concern, the outgoing message sent over an edge is simply evaluated by summing all messages received from the other edges. With regard to function nodes, let first consider the $C(f)$ nodes and focus on generic subchannel $\ell$. The square node corresponding to $C(\ell)$ is connected to all the variable nodes $x_{n,\ell}$ with $n \in \mathcal{N}(\ell)$. If we consider without loss of generality the message to be delivered to $x_{j,\ell}$ $(j \in \mathcal{N}(\ell))$, the $q$-th element of the return message is the solution of the following minimization
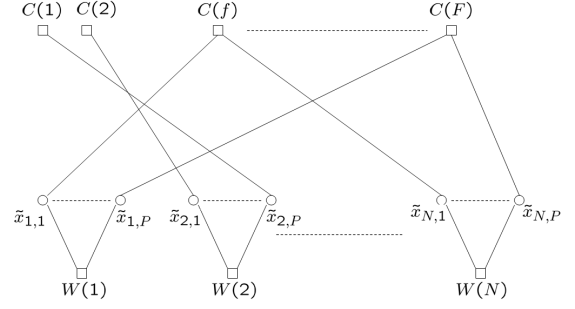
---

[3]See [7], [9] for a detailed description of MP algorithm for the sum-product problem.



Fig. 1. Factor graph for RRM. For ease of representation, we denote by $\tilde{x}_{n,p}$ $(p = 1, \ldots, P)$ the transmit format for user $n$ on the p-th ordered element of $\mathcal{P}_n$.

problem:

$$m_{j,\ell}^{(CV)}(q) = \min \sum_{n \in \mathcal{N}(\ell), n \neq j} m_{n,\ell}^{(VC)}(x_{n,\ell})$$
$$\text{subject to} \tag{11}$$
$$\sum_{n \in \mathcal{N}(\ell), n \neq j} \mathcal{I}(x_{n,\ell}) + \mathcal{I}(q) \leq 1$$

In a similar way, we can rewrite message passing rule for $W(n)$ nodes. Let focus on generic user $u$, the square node corresponding to $W(u)$ is connected to the variable nodes $x_{u,f}$ with $f \in \mathcal{P}_u$. If we consider without loss of generality the message to be delivered to $x_{u,\nu}$ $(\nu \in \mathcal{P}_u)$, the $q$-th element of the return message is the solution of the following minimization problem:

$$m_{u,\nu}^{(WV)}(q) = \min \sum_{f \in \mathcal{P}_u} P_{u,f}(x_{u,f}) + m_{u,f}^{(VW)}(x_{u,f})$$
$$\text{subject to}$$
$$\sum_{f \in \mathcal{P}_u, f \neq \nu} B\eta_{x_{u,f}} + B\eta_q \geq b_u \tag{12}$$
$$\sum_{f \in \mathcal{P}_u, f \neq \nu} P_{u,f}(x_{u,f}) + P_{u,\nu}(q) \leq P_{max,u}$$

When a message has been sent in both directions along every edge the algorithm stops. It is worth noting that in the considered OFDMA cellular scenario the $W(n)$ function node and its connected variable nodes are located at the $n$-th user, while all $C(f)$ function nodes are located at the BS. Hence, sending messages from variable nodes to $C(f)$ function nodes and vice-versa requires actual transmission on the radio channel. Instead, message exchange between variable nodes and $W(n)$ function nodes is performed locally at the users' terminals, without any transmission.

The solution of Problem (12) requires by far the largest computational effort, since it calls for an exhaustive search over all possible combinations of transmission formats. Thus, in the following we present a new formulation of (12) to find the optimal solution with limited complexity.

## B. A Dynamic programming algorithm

Given a user $u$, Problem (12) basically consists in finding a set of subchannels $f \in \mathcal{P}_u$, and for each selected subchannel

the related transmission format to use by $u$, so that a given function is minimized. Such problem can formulated as an Integer Linear Programming (ILP) problem introducing binary variables $y_{f,h}$ equal to 1 if the user transmits on the subchannel $f$ with the format $h$, and 0 otherwise. In a general form, such a problem can be rewritten as

$$
\begin{aligned}
\min \sum_{f,h} c_{f,h} y_{f,h} \\
\sum_{f,h} B\eta_h y_{f,h} &\geq \beta \\
\sum_{f,h,f\neq\nu} P_{u,f}(h) y_{f,h} &\leq \alpha \qquad (13)\\
\sum_h y_{f,h} &\leq 1 \quad f \in \mathcal{P}_u \\
y_{f,h} \in \{0,1\}
\end{aligned}
$$

where the cost $c_{f,h}$ is the cost for user $u$ of transmitting with format $h$ on subchannel $f$ (i.e., $c_{f,h} = P_{u,f}(h) + m_{u,f}^{(VW)}(h)$), $\beta = b_u - B\eta_q$, $\alpha = P_{max,u} - P_{u,\nu}(q)$. As in (12), the first two constraints correspond to the requirements on the bit-rate $b_u$ and on the maximal transmission power $P_{max,u}$, respectively. The subsequent $P$ constraints impose that at most one format is selected for each subchannel $f$. Note that, $b_u$ is limited from above by $B\eta_Q P$, and such a value is obtained when user $u$ transmits with format $Q$ on all the subchannels in $P_u$. Assuming that all admissible formats are multiple integer of a given spectral efficiency $\tilde{\eta}$, ie $\eta_h = h\tilde{\eta}$ ($h = 0, \dots, Q$), we can divide all terms of the first constraint of Problem (13) by $B\tilde{\eta}$ to obtain the equivalent constraint

$$
\sum_{f,h} h\, y_{f,h} \geq \frac{\beta}{B\tilde{\eta}} \qquad (14)
$$

where $\frac{\beta}{B\tilde{\eta}}$ is limited from above by $QP$. Observe that, the coefficients $h$ in the left-hand side of constraint (14) are integer values. Hence, since the left-hand side of (14) is integer, for any choice of variables $y_{f,h} \in \{0,1\}$, $\frac{\beta}{B\tilde{\eta}}$ can be rounded to $\lfloor \frac{\beta}{B\tilde{\eta}} \rfloor$. Moreover, we may assume that values $P_{u,f}(h)$ and $\alpha$ are integer (e.g., by multiplying all terms of the second constraint of Problem (13) by a suitable large number).

In the following, we show that Problem (13) can be solved by a *dynamic programming* approach [10]. Let $z_p(d,k)$ be the optimal solution value of Problem (13) defined on the first $p$ subchannels, with a "bit-rate" of $\lfloor \frac{\beta}{B\tilde{\eta}} \rfloor = d$ and a restricted maximal transmission power of $k$. We assume that $z_p(d,k) = +\infty$ if no feasible solution exists. Initially we set $z_0(0,k) = 0$ and $z_0(d,k) = +\infty$ for all $d = 1, \dots, QP$ and $k = 0, 1, \dots, \alpha$. To compute $z_p(d,k)$, we can use the recursion (15), where we assume that the minimum operator returns $+\infty$ if we are minimizing over an empty set. An optimal solution of Problem (13) can be found computing $z_P(QP, \alpha)$, and choosing the minimum of $z_P(j, \alpha)$ for $j = \lfloor \frac{\beta}{B\tilde{\eta}} \rfloor, \dots, QP$. The formula (15) requires the comparison of $Q$ terms, and, hence, the optimal solution value of Problem (13) can be found in $O(P^2 Q^2 \alpha)$ operations, only pseudopolynomial, since

$\alpha$ depends on the input data (i.e, $P_{max,u}$ and $P_{u,\nu}(q)$). Observe that, if no requirement is given on the maximum transmission power used by each user, i.e., if the constraint on $\alpha$ can be relaxed, Problem (13) can be solved in $O(P^2 Q^2)$ operations, polynomial in the number of subchannels in $\mathcal{P}_u$ and transmission formats.

### C. MP scheduling and Peeling procedure

As in traditional MP approach for the sum-product problem, if the factor graph is a tree there is a natural schedule for MP given by starting at the leaf nodes and sending a message once all incoming messages required for the computation have arrived [11]. Unfortunately, in general the graph which represents minimization problem (10) is not a tree (e.g., in Fig. 1 we have a cycle given by the path $\tilde{x}_{1,1}, C(f), \tilde{x}_{N,1}, \tilde{x}_{N,P}, C(F), \tilde{x}_{1,P}, \tilde{x}_{1,1}$). In this case, to completely define the algorithm for a generic factor graph we need to specify a schedule. It is worth noting that, even if message passing in the presence of cycles is strictly suboptimal, the solution found by means of iterative approaches is in most cases very close to the optimum (e.g., in the case of bitwise MAP decoding of linear block codes) [12],[13].

Iterative MP starts at variable nodes, which send an all zero message $\mathbf{m}_{n,f}^{(VW)} = 0$ to their adjacent $W(n)$ function nodes and then the algorithm proceeds in iterations. The pseudocode of Algorithm 1 illustrates the iterative MP algorithm for a generic user $n$. After $I$ iterations, each user peels off all variable nodes $x_{n,f}^{(I)} > 0$. All these variable nodes, say it *fulfilled nodes*, send a message to the BS to communicate that the corresponding subchannels have been reserved and the BS signals it to all users via the downlink broadcast channel.

---

**Algorithm 1** Iterative MP procedure for user $n$

---

**while** $\sum_{f \in \mathcal{P}_n} B\eta_{x_{n,f}} < b_n$ **do**
    $\mathbf{m}_{n,f}^{(VW)} \leftarrow 0$ $(f \in \mathcal{P}_n)$
    send $\mathbf{m}_{n,f}^{(VW)}$ to $W(n)$ $(f \in \mathcal{P}_n)$

    **for** $iter = 0$ to $I$ **do**
        evaluates $\mathbf{m}_{n,f}^{(WV)}$ according to (12) $(f \in \mathcal{P}_n)$
        $\mathbf{m}_{n,f}^{(VC)} \leftarrow \mathbf{m}_{n,f}^{(WV)}$ $(f \in \mathcal{P}_n)$
        send $\mathbf{m}_{n,f}^{(VC)}$ to $C(f)$ $(f \in \mathcal{P}_n)$

        **while** not received all $\mathbf{m}_{n,f}^{(CV)}$ from $C(f)$ $(f \in \mathcal{P}_n)$ **do**
            wait
        **end while**
        $\mathbf{m}_{n,f}^{(VW)} \leftarrow \mathbf{m}_{n,f}^{(CV)}$ $(f \in \mathcal{P}_n)$
    **end for**
    $\mathbf{m}_{n,f} \leftarrow \mathbf{m}_{n,f}^{(CV)} + \mathbf{m}_{n,f}^{(WV)}$ $(f \in \mathcal{P}_n)$
    $x_{n,f} \leftarrow \arg\min_{q=0,1,\dots,Q} \mathbf{m}_{n,f}(q)$ $(f \in \mathcal{P}_n)$
    Sends a message containing assignments $x_{n,f}$ to the BS
**end while**

---

Variables corresponding to fulfilled nodes are fixed and do not pass any message anymore. At this stage, all users evaluate wether they fulfill their rate constraints or not. Those users that satisfy their constraints stop participating to MP. All other users take part to successive iterations of MP, after having

$$z_p(d,k) = \min \begin{cases} z_{p-1}(d,k) + c_{p,0} & \text{(subchannel } p \text{ is not used by the user)} \\ z_{p-1}(d-1, k - P_{u,p}(1)) + c_{p,1} & \text{if } d-1 \geq 0 \text{ and } k - P_{u,p}(1) \geq 0 \text{ (}p\text{ is used with format 1)} \\ \dots \\ z_{p-1}(d-Q, k - P_{u,p}(Q)) + c_{p,Q} & \text{if } d-Q \geq 0 \text{ and } k - P_{u,p}(Q) \geq 0 \text{ (}p\text{ is used with format } Q) \end{cases} \quad (15)$$

updated their rate constraints in (12) on the base of the amount of resources they have been allocated. Before starting a new cycle of $I$ iterations, each user computes again the set $\mathcal{P}$ of the best $P$ subchannels among all subchannels which have not been yet assigned to other users. The process continues until the rate constraint is fulfilled.

## IV. NUMERICAL RESULTS

In this section we present the numerical results of the proposed algorithm. We have considered an hexagonal cell of radius $R = 500$ m. The uplink bandwidth is $W = 5$ MHz so the sampling time is $T_c = 200$ ns. Channel attenuation is due to path loss, proportional to the distance between the BS and the MS, and fading. The path loss exponent is $\alpha = 4$. We consider a population of data users with very limited mobility so that the channel coherence time can be assumed very long. The propagation channel is frequency-selective Rayleigh fading. The power of the $j$-th path is: $\sigma_j^2 = \sigma_h^2 \exp\left(-\frac{j}{\sigma_n}\right)$, $(j = 1, \dots, N_j)$ where $\sigma_h^2$ is a normalization factor chosen such that the average power of the channel is normalized to the value of the path loss, $\sigma_n = \sigma_\tau / T_c$ is the normalized delay spread with $\sigma_\tau = 0.5$ $\mu s$ and $N_j = \lfloor 3\sigma_n \rfloor$ is the number of paths taken into an account.

The available bandwidth $W$ is divided in $F = 32$ subchannels and there are $N$ active users at one time. We assume that all users request the same rate i.e., $b_n = b_0$, $(n = 1, \dots, N)$, so that $b_0 = W\eta_{avg}/N$, where $\eta_{avg}$ is the average spectral efficiency in the cell. The results shown in the following have been obtained by setting $\eta_{avg} = 1$ b/s/Hz and averaging on 500 channel realizations. We compare the performance of the proposed MP algorithm with two other resource allocation strategies:

1) The heuristic algorithm presented in [3] that we have indicated with the acronym BRCG (Babs + RCG) that solves the problem (2) by dividing it in three subproblems: 1) Decide the number of subcarriers each user gets based on rate requirements and the users average channel gain (bandwidth assignment based on SNR, BABS); 2) Select which subcarriers to allocate to each user according a greedy strategy (rate craving greedy, RCG); 3) Set the modulation for each subcarriers by employing a single-user bit loading technique.
2) A linear programming (LP) implementation of the allocation problem (2) formulated as in [4] where the rate constraints are translated into a number of subchannels to assign to each user. In our implementation, we set a unique transmission format for all users on all subchannels, so that each user is assigned the same number of subchannels $F/N$ and transmits with spectral efficiency

$\eta = \eta_{avg}$. By doing so, we neglect on purpose the impact of frequency diversity to focus only on the impact of multi-user diversity on the allocation performance.

As far as MP and BRCG are of concern, we set $Q = 4$, i.e., we consider four different transmission formats. Fig. 2 shows the average total transmitting power for different number of users. As far as MP parameters are of concern, we set $P = 4, 8, 12, 24$ for $N = 2, 4, 8, 16$, respectively. Moreover, since both BRCG and LP do not take into account any constraint on transmitting power, we set maximum transmitting power to $+\infty$ in (12). Note that the proposed MP algorithm requires the minimum average power in all cases. In particular, for small values of $N$, MP and BRCG clearly outperform LP, whilst for high values of $N$, MP and LP outperform BRCG. With few users, i.e., for small values of $N$, each user is assigned a great number of subchannels. In this case, the use of multiple transmission formats in MP and BRCG algorithms allows the transmitter to concentrate the power on the *best* channels while turning off the *worst* ones. Although designed to achieve an optimal global subchannel allocation, the LP scheme shows poor performance since it is forced to use the same transmission format over all assigned subchannels. On the other hand, increasing the number of users reduces the number of channels allocated and each user is assigned only 'good' channels, thus exploiting the so-called multi-user diversity. Fig. 2 shows that for all three algorithms increasing the number of users determines a reduction of the average transmission power. However, the BRCG algorithm can not fully exploit multi-user diversity, since greedy channel assignments are sub-optimal and, even using several transmission formats, is outperformed by the LP scheme already for $N \geq 8$.

Similar considerations can be drawn when considering the outage probability curves, i.e., Figs. 3-4. The outage probability $P_o$ is evaluated by specifying a maximum allowable transmitting power $P_{max}$ for each user. Such a maximum power is included in (12), so that outage events in the MP case occur when the iterative MP algorithm is not able to provide a feasible solution for all users. Differently, since in the LP and BRCG approaches we have not included constraints on the maximum transmitting power, outage events occur when, after the allocation, the power transmitted by a user exceeds $P_{max}$. Figs. 3-4 show $P_o$ as a function of $P_{max}$ for $N = 2$ and $N = 16$. The proposed MP scheme achieves the lower outage probability in all cases, thus confirming that it allows to perform an optimal subchannel assignment and to profitably exploit both frequency and multi-user diversity. Furthermore, although the computational complexity of MP depends on the number of iterations, at each iteration, the computation is naturally distributed among the transmitters, which have to
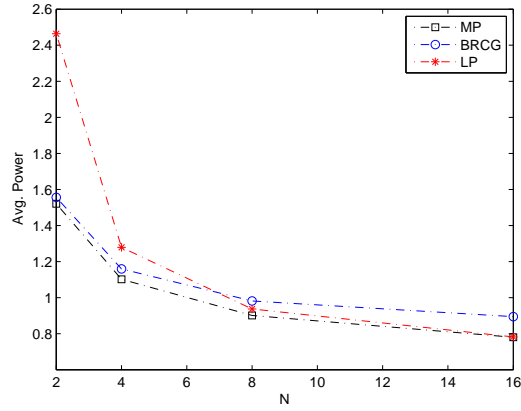
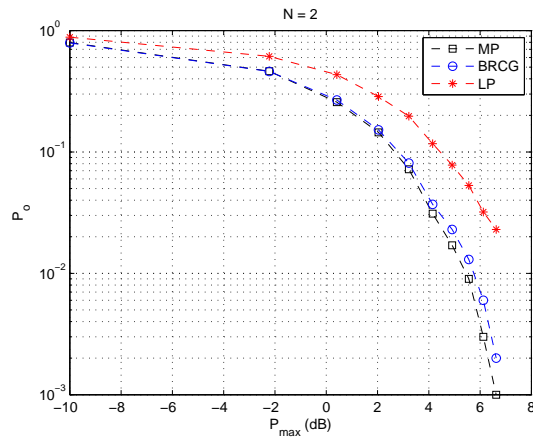Fig. 2. Average power consumption versus number of users.


Fig. 4. Outage probability versus maximum transmitting power for $N = 16$.


Fig. 3. Outage probability versus maximum transmitting power for $N = 2$.

allocation problems, such as global resource optimization in multi-cell OFMA systems.

## REFERENCES

[1] T. Keller and L. Hanzo, "Adaptive Modulation Techniques for Duplex OFDM Transmissions," *IEEE Trans. Veh. Techn.,* Vol. 49, 1893-1906, 2000

[2] R. Cheng, and S. Verdu, " Gaussian multiaccess channels with ISI: capacity region and multiuser water-filling," *IEEE Trans. Inf. Theory,* Vol. 39, 773-785, 1993.

[3] D. Kivanc, G. Li, and H. Liu, "Computationally Efficient Bandwidth Allocation and Power Control for OFDMA," *IEEE Trans. Wireless Comm.,* Vol. 2, 1150-1158, 2003.

[4] I. Kim, I. Park, and Y. Lee, "Use of Linear Programming for Dynamic Subcarrier and Bit Allocation in MultiUser OFDM," *IEEE Trans. Vehic. Technol.,* Vol. 55, 1195-1207, 2006.

[5] W. Rhee, and J. Cioffi, "Increase in capacity of multiuser OFDM System Using Dynamic Subchannel Allocation," *Proc. IEEE VTC 2000 Spring* 1085-1089, 2000.

[6] J. Jang, and K.B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journ. Sel. Areas in Commun.,* Vol. 21, pp. 171-178, 2003.

[7] T. Richardson, R. Urbancke, "Modern Coding Theory," *Cambridge University Press,* United Kingdom, 2008.

[8] Garey M. R., D. S. Johnson, "Computers and Intractability: A Guide to the Theory of NP-Completeness," *Freeman,* New York, 1979.

[9] David J. C. Mackay, "Information Theory, Inference, and Learning Algorithms" *Cambridge University Press,* United Kingdom, 2003.

[10] R.E. Bellman, "Dynamic Programming," *Princeton University Press,* 1957.

[11] F.R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform Theory,* Vol 47, No. 2, Feb. 2001.

[12] S. L. Litsyn and V. S. Shevelev, "On ensembles of low-density parity-check codes: asymptotic distance distributions," *IEEE Trans. Inform Theory,* Vol 48, No. 4, April 2002.

[13] S. L. Litsyn and V. S. Shevelev, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform Theory,* Vol 49, No. 12, Dec. 2003.

solve low-complexity and in practice small problems.

## V. CONCLUSION

We have proposed a novel distributed resource allocation scheme for the up-link of a cellular multi-carrier system based on the message passing (MP) algorithm. Resource allocation may rely on a similar MP procedure: with the goal of achieving a global optimal assignment, each transmitter iteratively sends and receives information messages to/from the base station until an allocation decision is taken. The exchanged messages are the solution of small distributed allocation problems. To reduce the computational load, the MP problems at the terminals follow a dynamic programming formulation. Hence, even if the computational complexity of MP depends on the number of iterations, at each iteration, the computation is naturally distributed among the transmitters, which have to solve low-complexity and in practice small problems. Numerical results show that the proposed approach is an excellent solution to the resource allocation problem for a single-cell multi-carrier system. Moreover, the distributed nature of the proposed strategy make it naturall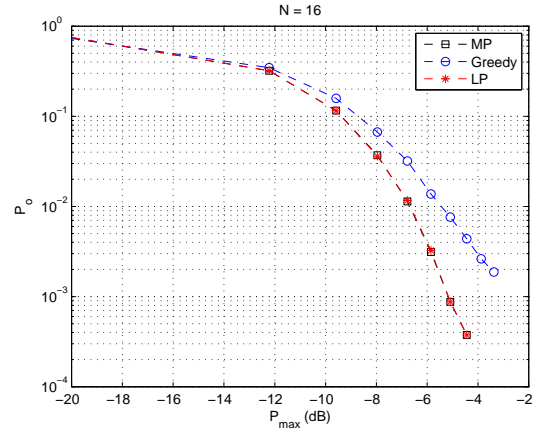y suitable for larger scale resource